SC 472
DESIGN OF BRIDGES

HYDRAULIC DESIGN CONSIDERATIONS

Lecture No. 4

Hydraulics as applied to highway drainage, is primarily involved with how to provide efficient and safe transport of water so as to avoid danger to properties, highway drainage structures and vehicles**.** For this reason, the first step in the design process is to determine the amount of water run-off which will be utilised in determining the size of the structure. The **run-off** is the amount of water which can flow at peak rain periods. For watersheds less than 2.59 km^2 (1 mile^2) in size, and for road-way or street drainage, the Rational Method is applied in determining the peak water run-off, ***Q*** in litre/sec. The equation defining the run-off, ***Q***, is expressed as:

$$\boldsymbol{Q = C.i.A} \qquad \text{.....(1)}$$

where *Q* = peak rate run-off

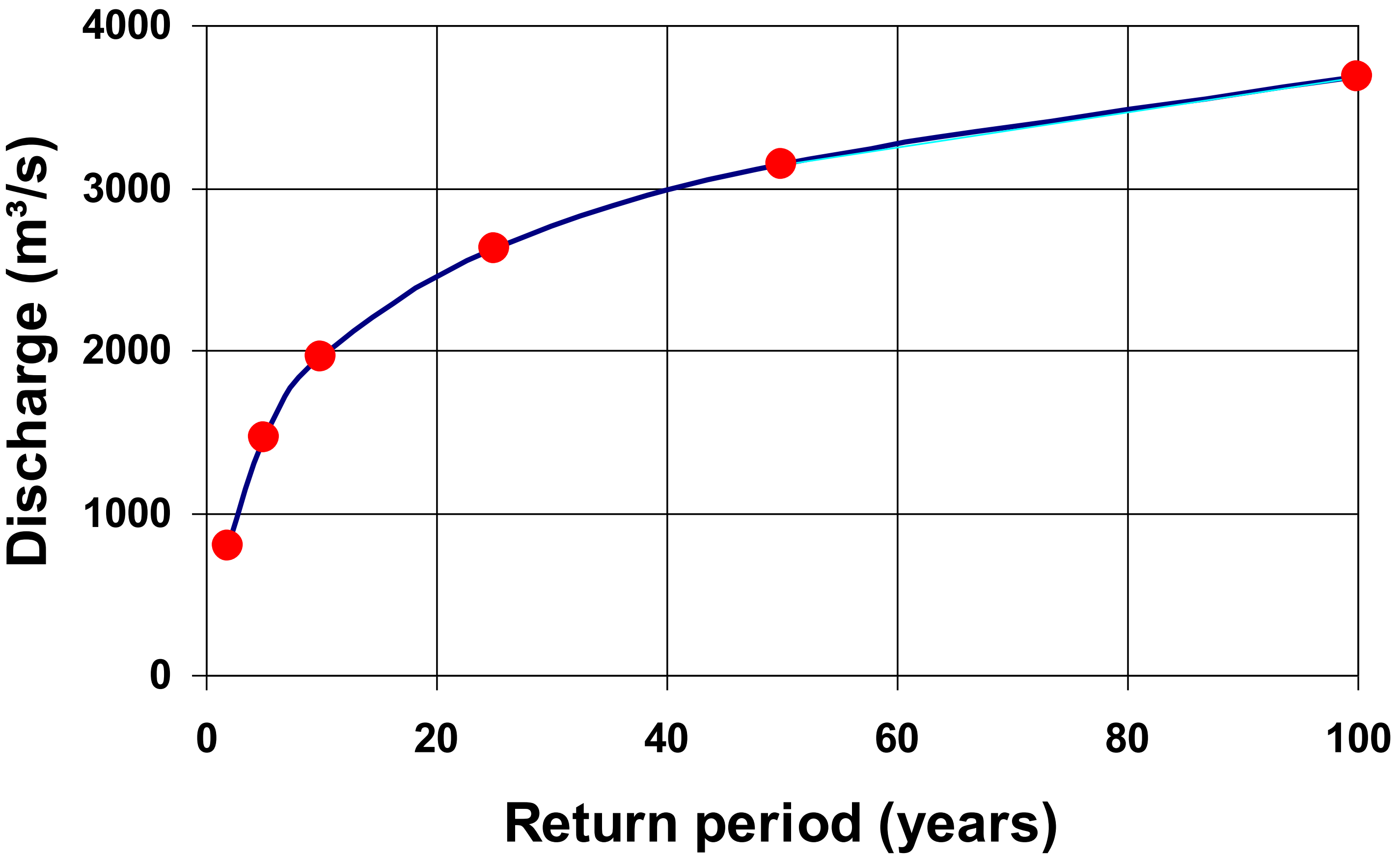
C **= run-off coefficient varying from 0.20 for heavily vegetated area to 0.90 for paved areas**

i = rainfall intensity (mm/min) for the time of concentration, t in minutes (i = 1mm/min = 166.67 l/(s.ha) Schneider (**1992)**

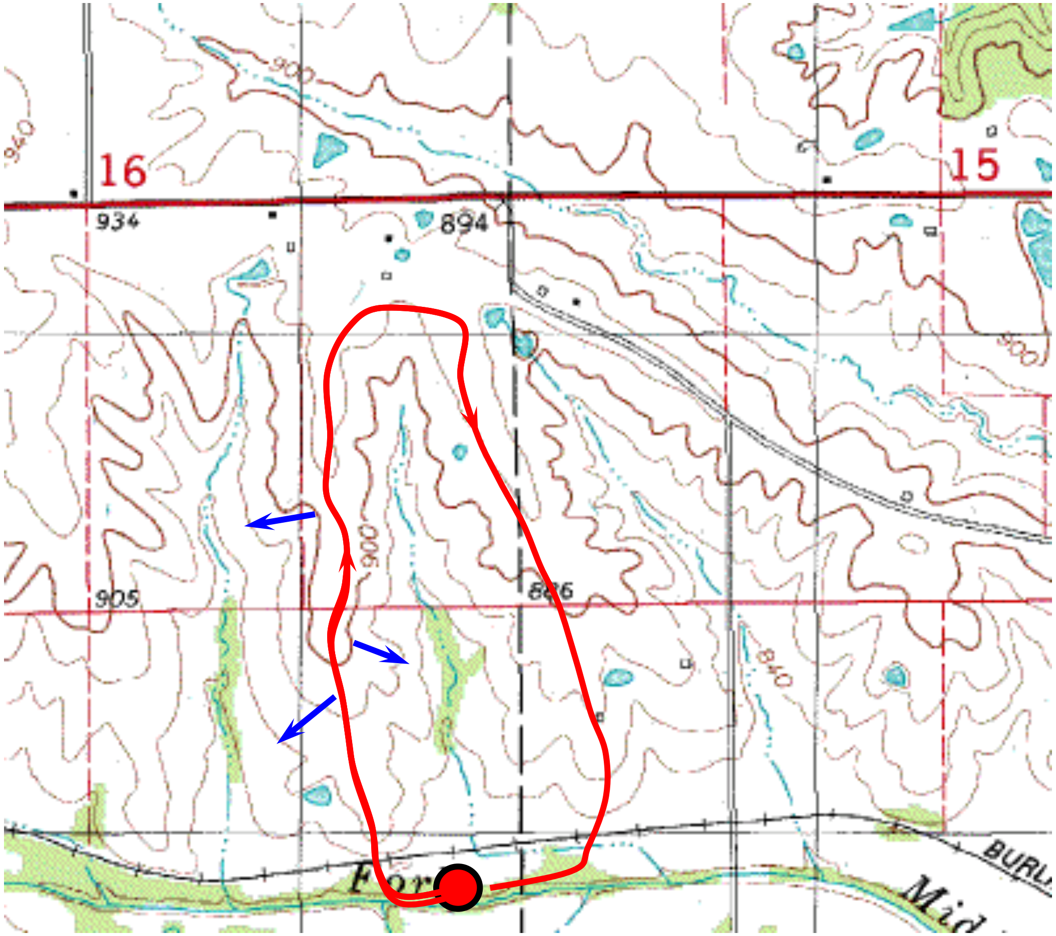
A = catchment (watershed) area in hactre

If the drainage structure (bridge or culvert) is not full flowing, its size can be determined by Manning's equation (see Eqn. 2a and 2b), using the relationship of the quantity of water that will flow through the structure, the velocity of water in the bridge and the area of the bridge opening as defined in the following sections;

Flood-Frequency Rating Curve



Drainage Area Delineation



$$Q = V.A_w \qquad \text{....(2a)}$$

in which

$$V = \frac{k}{n}.R^{2/3}.S^{1/2} \qquad \text{....(2b)}$$

where

Q = amount of water
V = velocity of water
R = hydraulic radius (Aw/P)
$\boldsymbol{A_w}$ = area of the section of water
S = slope of the culvert
n = Manning's coefficient
k = 1.486 for Imperial units and 1 for SI units = unit conversion factor

On reworking equations (2), the clear height of the drainage structure can be expressed as:

$$h = \left[\frac{Q^{3/2}}{\left\{ \frac{k}{n} . S^{1/2} \right\}^{3/2}} . \frac{(a + 2)}{a^{5/2}} \right]^{1/4} \qquad \text{....}(\mathbf{3})$$

where **a = h/L**

If a; **(h/L) = 1**, then equation (**3**) can take the form of:

$$h = \left[\frac{3.Q^{3/2}}{\left\{ \frac{k}{n} . S^{1/2} \right\}^{3/2}} \right]^{1/4} \qquad \text{....}(\mathbf{4})$$

Allowance should be made for freeboard in the obtained height, ***h***. According to Barfuss et al(**1994**), the Manning's coefficients for different types of material are as shown in Table 3.1.

Table 3.1: Manning's friction coefficients for different materials

Material type	**Manning's coefficient**
Clay tile	0.014
Brickwork	0.015
Masonry	0.025
Finished concrete	0.012
Unfinished concrete	0.014
Gravel	0.029
Earth	0.025

Dimensions

If the obtained **h** is the height of water just before the bridge is about to experience a full flow, it can be approximated that H = h, the minimum clear height of the bridge. Therefore the area of the bridge orifice will be as defined by equation (5) :

$$A_{cl} \square L.H \qquad \text{..........(5)}$$

Where ***L*** = minimum span of the bridge
H = minimum height of the bridge

The dimensions **H and L** have to be adjusted to provide reserve area for the unforeseen amount of water run-off especially during high floods.

River Hydraulics for Bridge Sizing

The geometry of a river channel is affected by the discharge, the characteristics of the bed and bank materials, and the sediment transport capacity of the channels. Depending on the type of river, there are different calculation approaches respectively as follows:

3.1 Sand Bed Channels

For sand bed channel, according to Blench, the mean channel width is given by the following Equations:

$$\boldsymbol{B \square 14Q^{0.5} D_{50}^{0.25} F_g^{-0.5}}$$

$$\boldsymbol{y \square 0.38q^{0.67} D_{50}^{-0.17}}$$

where B = mean channel width in m

y = mean depth of flow in m

Q = discharge in cumecs

q = discharge per meter width

D_{50} = medium size of the bed material in m

F_g = bank roughness

$F_g = 0.1$ for sand loam, $F_g = 0.2$ for silty clay loam

$F_g = 0.3$ for cohesive banks

3.2 Gravel Bed Channels

For gravel bed channels, the following equations are applicable;

$$B \square 3.26Q^{0.5}$$

$$y \square 0.47q^{0.8}D_{90}^{-0.12}$$

where D_{90} is the size of the bed material in m, other parameters are the same as above.

3.3 Cohesive Bed Soils

The resistance to scour of cohesive material is more complex than that of cohesionless materials. The only fairly reliable method of estimating scour is to measure soil properties and carryout model tests in the laboratory. The depth of the flow in channel may be calculated assuming that scour continues to occur till the tractive stress approaches the critical value. Thus:

$$y \square 51.4n^{0.86}q^{0.86}\tau_c^{-0.43}$$

where
- y = mean depth of flow in m
- n = Mannings coefficient
- q = discharge per meter width
- τ_c = the tractive stress for scour to scour in N/m^2

Soil type	*Critical tractive stress (N/mm²)*			
	Void ratio			
	2-1.2	1.2-0.6	0.6-0.3	0.3 – 0.2
Sand clay	1.9	7.5	15.7	30.2
Heavy clay	1.5	6.7	14.6	27.0
Clay	1.2	5.9	13.5	25.4
Lean clay	1.0	4.6	10.2	26.8

3.4 Effect of Bridge on River Regime

The construction of a bridge across a river or flood plain affects both the flow pattern and flow intensity, which may lead to local geometry change and new relationship between water and discharge.

Abutments and piers reduce the water way, increase the discharge, scour and head loss through the bridge openings. Protective works in the flood plain (wing walls, returns, river training works, etc) interfere with natural drainage and divert the flow from the flood plain to adjacent lands.

3.5 Linear Waterways

3.5.1 Stream with Rigid Boundaries - streams whose both banks and the bed are very rigid

When the banks and the beds of a stream are very rigid, the waterways of the bridge should be made equal to the width of the water surface measured from edge to edge along the design high flood level on the plotted section. However, a certain reduction in the waterway may be possible provided the velocity under the bridge is not severe, thus resulting in savings in the cost of construction of the bridge.

3.5.2 Quasi-Alluvial Streams

These are streams flowing between banks which are made up of rigid rocks or mixture of sand and clay, but the bed material is composed of loose granular material which can be picked up by the current and be transported away.

In this type of rivers, the waterway should be made equal the width of the water surface measured from edge to edge along the design high flood level.

3.5.3 Alluvial Streams – streams having erodible banks and erodible beds.

The linear waterway of a bridge across a fully alluvial stream should be kept equal to the regime width as given by Lacey in the following equation:

For a Regime cross section:

$$P = 4.8Q^{0.5}$$

$$R = 0.473\left(\frac{Q}{f}\right)^{1/3}$$

$$A = 2.3\frac{Q^{5/6}}{f^{1/3}}$$

Regime velocity and Slope

$$v = 0.44Q^{1/6}f^{1/3}$$

$$S = 0.0003\frac{f^{5/4}}{Q^{1/6}}$$

Regime width and depth

$$w = 4.8\sqrt{Q}$$

$$d = 0.473\left(\frac{Q}{f}\right)^{1/3}$$

Silt factor, $f = 1.76\sqrt{m}$

where

Q	=	discharge in cumecs
P	=	wetted perimeter in m
R	=	hydraulic mean depth in m
A	=	cross sectional area in sq.m
v	=	velocity of flow in m/s
w	=	regime width in m
d	=	regime depth in m
m	=	mean diameter of particles in m
f	=	silt factor

The value of f for different types of bed materials are given in table below

Types of bed material		*Grain size (mm)*	*f*
Silt			
	Very fine	0.0081	0.500
	Fine	0.1200	0.600
	Medium	0.2330	0.850
	Standard	0.3230	1.000
Sand			
	Medium	0.5050	1.250
	Course	0.7250	1.500

3.6 Economic Span

Economic span is one for which the total cost of the bridge is minimum. For the most economical span, the cost of the superstructure equals the cost of substructure, with the following assumptions:

- The cost of the superstructure is proportional to the square of the span
- The spans are of equal length
- The cost of each abutment is the same
- The cost of railings, parapets, approach is constant

Let

A be the cost of each abutment
B be the cost of each pier
C be the cost of railings, parapets
D be the cost of approach
T be the total cost of the bridge
N be the number of spans
L be the length of each span, and
Lt be the total span of the bridge

Therefore, the total cost of the bridge is

$$T = A + (N\text{-}1)B + C + D + NkL^2$$

Where k is the cost coefficient of the superstructure

For minimum cost $\dfrac{dT}{dL} \square 0$

Differentiating the above equation, w.r.to L and equating to zero, and by writing Lt/L = N, we get

$$B \square kL^2$$

hence the economic span (Le), the cost of the superstructure of one span is equal to the cost of the substructure of the same span, that is:

$$L_e \square \sqrt{\frac{B}{k}}$$

3.7 Afflux

It is rarely feasible economically to bridge the river in one span. Normally, piers are located within the main flow channel and embankments encroach into the flood plain. These obstruct the flow and cause the upstream water level to rise above the free discharge level. This heading up of water on the upstream side of the bridge is known as afflux. It is one of the important parameters required to fix the various levels for the bridge. The velocity of flow under the bridge is also governed by afflux. The vertical clearance and freeboard are influenced by afflux as well.

Freeboard

The vertical clearance is the difference between the high flood level (HFL) and the lowest point on the superstructure. The freeboard is the difference between the highest flood level after allowing for afflux if any, and the formation level of the communication route on the top level of the guide banks.

Some of the formulae used for computation of afflux are as follows:

Some of the formulae used for computation of afflux are as follows:

Melesworth formula $$x = \left(\frac{v^2}{17.9} + 0.015\right)\left(\frac{A^2}{a^2} - 1\right)$$

Where

x	=	afflux in m
v	=	the normal velocity of flow in m/s
A	=	the area of natural waterway in sq.m
a	=	the area of artificial waterway in sq.m

Marriman's formula $$x = \frac{v^2}{2g}\left[\left(\frac{A}{Ca}\right)^2 - \frac{A}{A_t}\right]$$

Where

g	=	acceleration due to gravity
A_t	=	the enlarged area of the upstream of the bridge in sq.m
C	=	$0.75 + 0.35(a/A) - 0.1(a/A)^2$

The definitions of x, v, a and A are the same as above.

Allowable velocity

As afflux causes an increase in the velocity of flow through the bridge, it is normally limited to 200 to 300 mm. The allowable safe velocities for different types of soils under the bridge are

Loose clay or fine sand	- up to 0.50 m/s
Coarse sand	- 0.50 to 1.00 m/s
Fine gravel, rocky soil	- 1.00 to 2.50 m/s
Boulder and rock	- 2.50 to 5.00 m/s

In case the velocity goes beyond the permissible safe limits, suitable protective works would be needed.

Scour

Scour occurs during the passage of high discharge, when the velocity of the stream exceeds the limiting velocity that can be withstood by the particles of the bed material.

For a straight reach of the stream and where the bridge in a single span structure;

Alluvial Stream

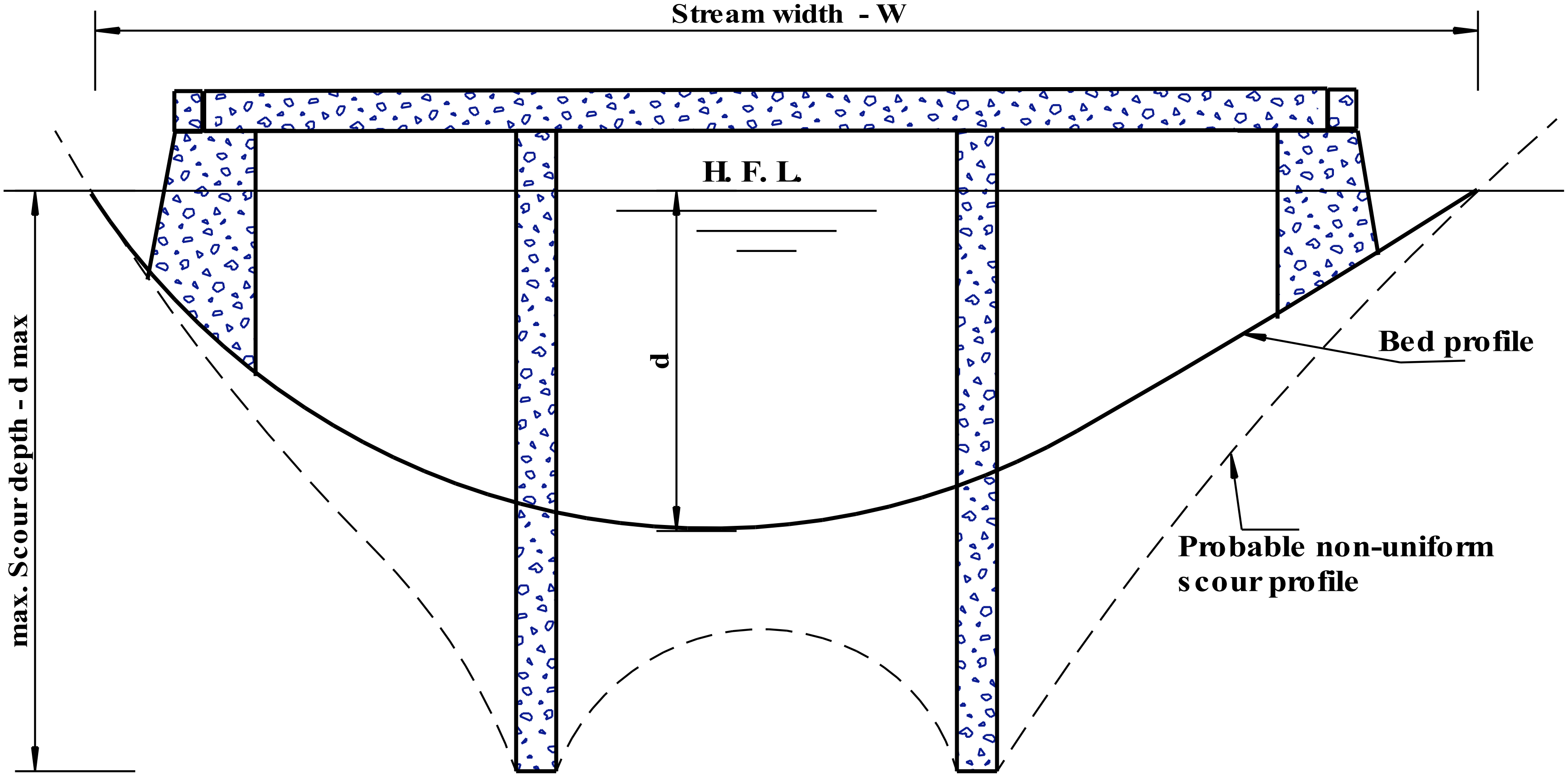
Normal scour depth $$d = 0.473\left(\frac{Q}{f}\right)^{1/3}$$

Where: Q = discharge in cumecs

f = silt factor

Maximum scour depth $d_{max} = 1.5d$, *and* For non-uniform scour $d_{max} = d\left(\frac{W}{L}\right)^{1.56}$

The larger of the two values obtained from the above equations is chosen as the maximum scour depth


Stream width - W
H. F. L.
d
max. Scour depth - d max
Bed profile
Probable non-uniform
scour profile

Quasi ALLUVIAL Streams

For narrow cross sections, the estimation of maximum scour depth has been explained in the preceding subsections. For wide rivers, one of the following equations is used to estimate the maximum scour depth:

Without constriction

Normal scour depth

(a) when the velocity is known: $d = \dfrac{Q}{vw}$

(b) when Q, N, S and W are known: $Q = \dfrac{wd^{5/3} S^{1/2}}{N}$

(c) when both velocity and slope are not known $d = \dfrac{1.21 Q^{0.63}}{f^{0.33} W^{0.60}}$

For non-uniform scour $d_{max} = d\left(\dfrac{W}{L}\right)^{1.56}$

Maximum scour depth $d_{max} = 1.50d$

The larger of the two values obtained above is chosen as the maximum scour depth

<u>With constriction</u>

Normal scour depth $$d' = d\left(\frac{W}{L}\right)^{0.61}$$

Maximum scour depth $$d_{max} = 1.5d'$$

For non-uniform scour $$d_{max} = d\left(\frac{W}{L}\right)^{1.56}$$

The larger of the two values obtained above is chosen as the maximum scour depth

Worked Examples

<u>Example 1</u>

Given that the flood discharge of a stream is 225m^3/s, the velocity of water is 1.5 m/s, and the width of flow at flood level is 60.0m. Determine the water way when the allowable velocity under the bridge is 1.8 m/s.

Solution

Area of natural waterway:

$$A = Q/v = 225/1.5 = 150\ m^2$$

Mean depth of flow:

$$d \square \frac{A}{L} \square \frac{150}{60} \square 2.50m$$

Area of artificial waterway:

$$a \square \frac{Q}{v_{al}} \square \frac{225}{1.8} \square 125m^2$$

Afflux from Molesworth formula,

$$x = \left(\frac{v^2}{17.9} + 0.015\right)\left(\frac{A^2}{a^2} - 1\right)$$

$$x = \left(\frac{1.5^2}{17.9} + 0.015\right)\left(\frac{150^2}{125^2} - 1\right) = 0.062m$$

Linear waterway

$$L_l = \frac{a}{d + x} = \frac{125}{2.5 + 0.062} = 48.79m$$

If you take 90% of the allowable velocity, *a* will be bigger than the one obtained above, the afflux will be smaller and the resulting linear waterway will be slightly larger than the above value.

Example 2

Design a waterway for a bridge over a trapezoidal channel having side slope of 1:1 with a discharge of 25 m^3/s, a bed fall of 1:1000 and bed width to depth ratio of 6:1. The bed material is sand with a safe velocity of 2.5m/s. The afflux should not be more than 8cm. take Manning coefficient, n = 0.025

Solution

Area of flow $$A = b \times d + 2\frac{d^2}{2}$$

$$= d(b + d) = 7d^2 \quad (\because b = 6d)$$

Wetted perimeter $$P = b + 2d\sqrt{2} = 8.883d$$

Hydraulic mean depth $$R = \frac{A}{P} = \frac{7d^2}{8.83d} = 0.80d$$

From Manning's formula we have

$$v = \frac{R^{2/3} S^{1/2}}{n} = \frac{(0.80d)^{2/3}\left(\frac{1}{1000}\right)^{1/2}}{0.025} = 1.1d^{2/3}$$

The flood discharge $Q = v.A$

Or $25 = 1.1d^{2/3}.7d^{2}$

Therefore, *d = 1.55m*

and P = 8.83d = 8.83 x 1.55 = 13.68 m

Hence $v = 1.1d^{2/3} = 1.1 \times (1.55)^{2/3} = 1.47 m/s$

Since the velocity under the bridge is less than the given safe velocity of 2.5m/s, the design is OK

According to Drown Weir formula

$$Afflux, \quad x = \frac{v^2 d^2}{2g(d + x)^2}\left(\frac{L^2}{C^2 L_l^2} - 1\right)$$

Average length L = 6d + 2d/2 = 10.85m

Therefore;

$$0.08 = \frac{1.47^2 \times 1.55^2}{2 \times 9.81(1.55 + 0.08)^2}\left(\frac{10.85^2}{0.95^2 L_l^2} - 1\right)$$

Hence, $L_l = 8.50m$

and

$$v = \frac{Q}{7d^2} = \frac{25}{7 \times 1.55^2} = 1.49m/s$$

Example 3

A bridge has a linear waterway of 120 m constructed across a stream whose natural waterway is 200m. If the flood discharge is 1000m^3/s and the mean depth of flow is 3 m, calculate the afflux under the bridge.

Solution

Area of the natural waterway $A = 200x3 = 600m^2$

Normal velocity of flow $v = \frac{1000}{600} = 1.67 m/s$

From Drown Weir formula, we have

$$x = \frac{1.67^2 \times 3^2}{2 \times 9.81(3+x)^2}\left(\frac{200^2}{0.95^2 \times 120^2} - 1\right)$$

or $$x^3 + 6x^2 + 9x - 2.65 = 0$$

By trial and error, the solution for the cubic equation is: *x = 0.252 m*

Hence, t**he Afflux** ***x = 0.252 m***

Example 4

The approximate costs of one superstructure and one pier for a multispan bridge are given below. Estimate the economic span.

Span (m)	12	18	21
Superstructure cost (US$)	34,000	80,000	150,000
Substructure cost (US$)	50,000	54,000	48,000

Solution

The average cost coefficient is calculated as shown in the following table. This calculation is based on the assumption that the cost of the superstructure is proportional to the square of the span.

Span			Cost coefficient
12m	$34{,}000/12^2$	=	236.1
18m	$80{,}000/18^2$	=	246.9
21m	$150{,}000/21^2$	=	340.0

Avg. cost coefficient, *k* = $\frac{1}{3}(236.1 + 246.9 + 340) = 274.3$

Average cost of one pier (US$), B = $\frac{1}{3}(50{,}000 + 54{,}000 + 48{,}000)$

B = 50,666

Economic span $$L_e = \left(\frac{B}{k}\right)^{0.5} = \left(\frac{50{,}666}{274.3}\right)^{0.5} = 13.6m$$

Hence, the economic span is 13.6 m

Example 5

The flood discharge under a bridge is 300m^3/s. If the river bed has a deep layer of coarse sand, determine the maximum depth of scour under piers and abutments. Take the silt factor for coarse sand, f = 1.5

Solution

Normal depth of scour

$$d = 0.473\left(\frac{Q}{f}\right)^{1/3}$$

$$d = 0.473\left(\frac{300}{1.5}\right)^{1/3} = 2.76m$$

Therefore,

Depth of scour under piers = ***2d*** = 2.76 x 2 = 5.52 m

Depth of scour under abutments = ***1.5d*** = 4.14 m.

Example 6

A two-span girder bridge is to be provided across a river having the following data:

Flood discharge	100 m^3/s
Bed width	30 m
Side slope	1:1
Bed level	50.00 m
HFL	52.50 m
Maximum allowable afflux:	15 cm

Task: Calculate the span of the bridge

Solution

Area of flow: ***A = 30 x 2.5 + 2(0.5 x 2.5 x 2.5) = 81.25 m^2*** --→ because the slope is 1:1

Normal velocity of flow: ***v = Q/A = 100/81.25 = 1.23 m/s***

From Molesworth Formula we have

$$x = \left(\frac{v^2}{17.9} + 0.015\right)\left(\frac{A^2}{a^2} - 1\right)$$

$$0.15 = \left(\frac{1.23^2}{17.9} + 0.015\right)\left(\frac{A^2}{a^2} - 1\right)$$

$2.5 = 81.25^2/a^2$,

hence ***$a = 51.38\ m^2$***

Span of the bridge

$$L_1 = \frac{a}{d + x} = \frac{51.38}{2.5 + 0.15} = 19.39m$$